Resolution and Accuracy of Cryogenic Temperature Measurements

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A procedure is outlined and typical data provided for calculation of achievable resolutions and accuracies using commercially available cryogenic temperature sensors suitable for use as secondary or tertiary standards. Differences between resolutions achievable in absolute temperature measurements as opposed to measurements of temperature changes are discussed. Methods for estimating or determining errors are discussed and typical sensor calibration errors are given.

INTRODUCTION

Temperature resolution and accuracy are important, but are not the only, considerations when choosing a temperature sensor and its associated measurement system. Other considerations include: sensor size or thermal mass, stability over time, response time, mechanical shock resistance, interchangeability, measurement system simplicity, cost, magnetic field effects, and resistance to ionizing radiation. The scope of this paper is limited to the estimation of resolutions and accuracies possible when making cryogenic temperature measurements with commercially available temperature sensors.

Cryogenic temperature sensors have been developed based on a variety of temperature-dependent properties (1). Common, commercially available sensors include resistors, capacitors, thermocouples, and semiconductor junction devices such as diodes or transistors. The temperature-dependent characteristics of such sensors are published elsewhere (2,3). Such sensors, suitable for use as a secondary or tertiary temperature standards, are of primary concern in this paper. Primary standards-grade sensors are very sensitive to thermal and mechanical shock and are therefore not suitable for ordinary laboratory or industrial temperature measurements. Other temperature measurement techniques such as gas, vapor pressure, acoustic, noise, and magnetic susceptibility thermometry, are not covered by this paper as they require much greater effort to implement or they severely constrain system design.

Temperature resolution is the smallest temperature change that can be detected. The precision (or reproducibility or stability) is a measure of how closely the measured values are grouped. Accuracy is indicated by the difference between measured and true values of a parameter. The accuracy of a single measurement can be no better than the resolution, but is degraded by calibration and measurement errors. The relevant equations for determining resolution and accuracy depend on whether the measurement is of the absolute temperature or of a temperature change. In either case, the achievable resolution depends on 1) the sensor characteristics and 2) the measurement system resolution. The accuracy of a temperature measurement can be evaluated using error analysis.

ABSOLUTE TEMPERATURE RESOLUTION

The temperature resolution $\mathcal{E}_T$ of a thermometer measuring a temperature T is limited by the measurement system resolution $\mathcal{E}_v$ according to the expression

$$\mathcal{E}_T = \frac{\mathcal{E}_V}{dV/dT} \qquad (1)$$

when the sensitivity dV/dT of the thermometer does not change significantly within $\mathcal{E}_T$ of the temperature, T. The measured parameter and the system resolution, V, are assumed to be voltages in Equation 1. The sensitivity dV/dT can be written as I(dR/dT) in the case of a ohmic resistance thermometer excited with a constant current, I. Equation 1 can be put in dimensionless form by dividing both sides by T and dividing the numerator and denominator of the right hand side by V, yielding

$$\frac{\mathcal{E}_T}{T} = \frac{\mathcal{E}_V / V}{(T/V)\,(dV/dT)} \equiv \frac{\mathcal{E}_{rel}}{S} \qquad (2)$$

The dimensionless group in the numerator is the relative measurement system resolution, $\mathcal{E}_{rel}$, consisting of the measurement system resolution, $\mathcal{E}_V$, divided by the voltage measured. The denominator consists of $S \equiv (T/V)(dV/dT)$, known as the specific sensitivity, giving the relative temperature sensitivity of the thermometer at temperature T. The specific sensitivity is also equal to (d ln$V$/d ln$T$), the slope of the parameter versus temperature on a log-log plot. Note that equations 1 and 2 can be made to apply to thermometers based on other temperature-dependent properties (*e.g.*, capacitance, resistance or pressure) by replacing V with C, R or P.

The dimensionless nature of Equation 2 makes somewhat easier the comparison of thermometers based on different temperature-dependent properties. Specific sensitivities of some representative cryogenic temperature sensors are plotted in Figure 1. Sensors of the same type made by different manufacturers may have similar characteristics. Non-metallic sensors of the same type but different nominal resistances usually have different S versus T characteristics. Metallic resistance thermometers should all fall on the same line with some exceptions: variations in residual resistance cause differences in specific sensitivity at lower temperatures, and the sensitivity of alloys such as Rh-Fe also depends on the concentration of the active impurity.

A specific sensitivity in the 0.1 to 10 range is usually best for temperature measurements over a wide range, although other factors can be much more important. A large specific sensitivity allows the resolution of small temperatures relative to the temperature measured, but the temperature range becomes limited if the value of the property measured becomes too large or small to be determined accurately with the measurement system.

The relative absolute temperature resolution is also a function of the relative measurement system resolution, $\mathcal{E}_{rel} = \mathcal{E}_V / V$ ($\mathcal{E}_C / C$ for capacitance measurements). As an example, a germanium resistance sensor with a specific sensitivity of -2.14 and resistance of 1000 Ω at a temperature of 4.2 K, 1 µA excitation current, and a measurement system with 1 µV resolution would provide an absolute temperature resolution of about 2 mK. Note that the sensor excitation current affects the output voltage *(V = IR),* and thus the relative measurement system resolution, so the sensor and the measurement system are not independent. Absolute temperature resolutions calculated using Equation 2, the specific sensitivities plotted in Figure 1, and sample excitations and system resolutions are plotted in Figure 2. The temperature resolutions plotted in Figure 2 were calculated only as a demonstration of how to calculate temperature resolutions for a variety of different sensors; different operating conditions, sensor models, or measurement equipment can greatly affect the achievable resolution.

Optimization of the temperature resolution is dependent on both the sensor properties and the measurement system. The minimum resolvable temperature is not merely a matter of finding a sensor with the highest specific sensitivity. Some examples of interactions between sensor properties and measurement system resolution follow.

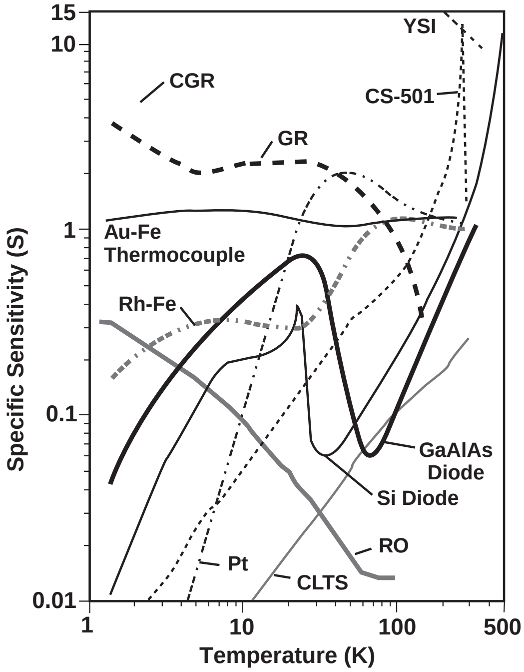

Figure 1. Absolute values of specific sensitivities of representative commercial cryogenic temperature sensors. Model numbers refer to Lake Shore sensors except where noted. **Au-Fe thermocouple:** KP chromel vs. Au-0.07%Fe, **CGR:** CGR-1-1000 carbon-glass resistor, **CLTS:** Vishay Micro-Measurements CLTS-2 metal foil gauge, **CS-501:** CS-501 capacitor, **GaAlAs diode:** TG-120P gallium-aluminum-arsenide @ 10 µA, **GR:** GR-200A-1000 germanium resistor, **Pt:** Pt-103 platinum resistor, **Rh-Fe:** RF-800-4 rhodium-iron resistor, **RO:** Scientific Instruments RO-600 ruthenium oxide sensor, **Si diode:** DT-470 @ 10 µA, **YSI:** Yellow Springs Instruments 44003A thermistor.

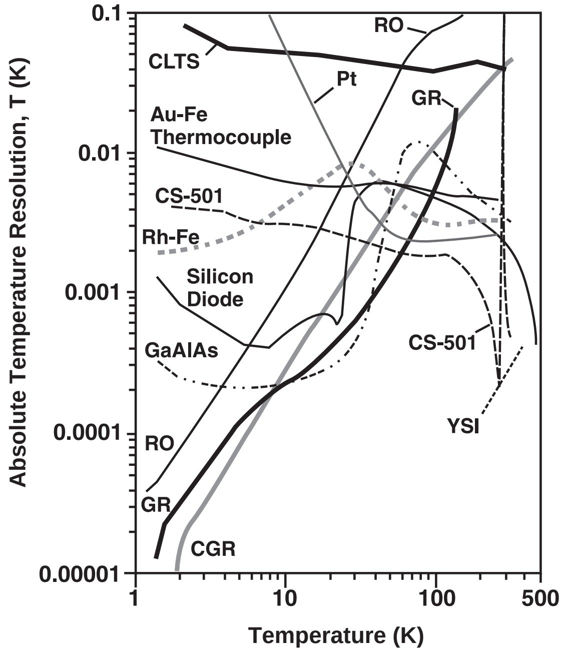

Figure 2. Absolute temperature resolutions of representative commercial cryogenic temperature sensors under the following operating conditions: **Au-Fe thermocouple:** versus KP chromel, **CGR:** 1-3 mV or I = 0.1 µA minimum, **CLTS:** 10 µA, **CS-501:** 5 kHz charging current, **GaAlAs diode:** 10 µA, **GR:** 1-3 mV or I = 0.1 µA minimum, **Pt:** 100 µA, **Rh-Fe:** 300 µA, **RO:** 10 µA, **Si diode:** 10 µA, **YSI:** 1 µW. Relative measurement system resolution: capacitance: $\mathcal{E}_{rel}$ = 0.1 pF/C; voltage: $\mathcal{E}_{rel}$ = 0.1 µV/V or 10^{-5}, whichever is larger. Refer to Figure 1 for sensor identification.

As a first example, gold-iron versus chromel thermocouples have what appears to be a nearly ideal specific sensitivity near unity across the entire 1 to 300 K temperature range. Unfortunately, thermocouples suffer from very small signal output, which can decrease the temperature resolution possible from a given measurement system. Thermocouples are also affected by nonuniformities in the wire and require a good understanding of thermocouple physics for proper installation and operation (4).

A sensor with large specific sensitivity, such as a germanium resistor near 1 K, can be limited in resolution by power dissipation constraints. The germanium crystal requires strain-free mounting for accurate temperature readings and long term stability, but the strain-free mounting reduces the thermal contact between the sensor and the body whose temperature is to be measured, making the sensor more susceptible to self heating. The excitation current for germanium and carbon-glass sensors is typically adjusted to produce an output voltage in the 1 to 3 mV range, thereby maintaining a balance between signal level and power dissipation. Other sensors such as platinum or thick film resistors do not require strain-free mounting, so signal levels of thin film or encapsulated platinum sensors can be increased by operating with higher power dissipation. The trade off is that strain-free mounted platinum sensors are more stable over time.

A diode is an example of a sensor that can have relatively low specific sensitivity, but large signal level, typically on the order of a volt. Potentials on the order of one volt can be measured with great resolution. Diodes, however, are non-ohmic and thus constrained to constant current operation, which can lead to self-heating problems at low temperatures.

Optimization of the absolute temperature resolution can require complex tradeoffs between sensor and measurement system costs and capabilities.

RELATIVE TEMPERATURE RESOLUTION

Better resolution is possible with the same measurement system when measuring temperature changes (relative temperatures) smaller than the absolute temperature. The reason for this fact is that only the change in the value, and not the entire value, must be measured. In this case, Equation 2 is not valid since the specific sensitivity is defined using the full parameter value (*e.g.*, V) whereas the relative system response requires the change in the measured value (*e.g.*, ΔV). Equation 1 is valid, but provides little guidance for optimizing the resolution of relative temperature measurements. Equation 2 can be modified to apply to relative temperature measurements by multiplying the right hand side by (ΔV/ΔV), yielding the expression

$$\frac{\mathcal{E}_T}{T} = \left(\frac{\Delta V}{V}\right)\frac{\mathcal{E}_V/\Delta V}{(T/V)(dV/dT)} \equiv \left(\frac{\Delta V}{V}\right)\frac{\mathcal{E}_{rel}}{S} \qquad (3)$$

The resolvable temperature is seen to be reduced by a factor of ($\Delta V/V$) if $\mathcal{E}_{\mathrm{rel}}$ remains the same as for an absolute temperature measurement. In practice, the system resolution, $\mathcal{E}_V$, is ordinarily not reduced in proportion to the ratio ($\Delta V/V$) so less resolution gain is realized. Note that both equations 1 and 3 implicitly or explicitly require knowledge of the absolute temperature, T (the sensitivity dV/dT at temperature T is required in Equation 1). This problem can be avoided by using a thermometer with a linear response to temperature. Alternately, the relative temperature can be measured with one thermometer while the absolute temperature is measured with a second thermometer, but the accuracy of the absolute temperature measurement will affect the accuracy of the relative temperature measurement.

SOURCES OF MEASUREMENT ERROR

Equations 1 - 3 can be used to calculate the temperature resolution (or error) once the measurement system resolution (or error) is specified. This section discusses the sources of the errors and how to determine their magnitudes. Error sources include the sensor calibration, the applied excitation, measurement system calibration, thermal voltages, noise, sensor self-heating and poor thermal grounding of the sensor.

The total error arising from several independent error sources is usually calculated in one of two ways. The worst-case error, ε_{WC}, can be estimated by direct summation of all errors

$$\varepsilon_{WC} = \varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_i + \cdots + \varepsilon_n \quad (4)$$

where ε_i is the i^{th} of n total errors.

The most probable error, ε_{MP}, can be estimated by assuming a statistical distribution of errors, in which case the errors are summed in quadrature according to

$$\varepsilon_{MP} = \sqrt{\varepsilon_1^2 + \varepsilon_2^2 + \cdots + \varepsilon_i^2 + \cdots + \varepsilon_n^2} \quad (5)$$

The worst-case and most probable errors must be computed from errors of the same dimensions. Dimensionless relative system errors can be summed using either Equations 4 or 5 and then translated to temperature errors using Equations 2 or 3.

Getting statistical data suitable for addition by quadrature can be a problem; instrument and sensor specifications commonly give maximum rather than most probable or typical values for errors. Two approaches may be taken to dealing with maximum error specifications. The conservative approach is to use the specification limit value in worst case or most probable error calculations. The less conservative approach is to assume a statistical distribution within the specification limits and assume the limit is roughly three standard deviations, in which case one-third of the specification limit is used in error calculations. The manufacturer may be able to supply additional information to help improve error estimates.

Voltage or Frequency Measurement Errors

The accuracy of instrumentation such as voltmeters and frequency counters is subject to calibration uncertainty and drift with time and operating temperature. Accuracies of such instruments should be available from the manufacturer.

Excitation Current Error

The temperature measurement error due to an error in the excitation current can be calculated from Equation 2 by replacing the quantity ε_V/V by the relative voltage change due to the current error. The resulting expression is

$$\frac{\varepsilon_T}{T} = \frac{(\varepsilon_I/I)(R_d/R_s)}{S} \quad (6)$$

where R_d and R_s are the dynamic and static resistances of the sensor. Note that the dynamic and static resistances of an ohmic sensor are equal. Typical dynamic resistances of a Lake Shore DT-470 silicon diode are 3000 Ω at 300 K, 1000 Ω at 77 K, and 2800 Ω at 4.2 K, while the static resistances are respectively 51.9 kΩ, 102 kΩ and 163 kΩ.

Thermal (Johnson) Noise

Thermal energy produces random motions of the charged particles within a body, giving rise to electrical noise. The minimum rms noise power available is given by $P_n = 4kT\Delta f_n$, where k is the Boltzmann constant and Δf_n is the noise bandwidth. Peak-to-peak noise is approximately five times greater than the rms noise. Metallic resistors approach this fundamental minimum, but other materials produce somewhat greater thermal noise. The noise power is related to current or voltage noise by the relations: $I = [P_n/R]^{1/2}$ and $V = [P_nR]^{1/2}$. The noise bandwidth is not necessarily the same as the signal bandwidth, but is approximately equal to the smallest of (5):

- π/2 times the upper 3 db frequency limit of the analog dc measuring circuitry, given as approximately $1/(4\,R_{eff}\,C_{in})$ where R_{eff} is the effective resistance across the measuring instrument (including the instrument's input impedance in parallel with the sensor resistance and wiring) and C_{in} is the total capacitance shunting the input;
- $0.55/t_r$ where t_r is the instrument's 10-90% rise time;
- one Hz if an analog panel meter is used for readout; or
- one-half the conversion rate (readings per second) of an integrating digital voltmeter.

Thermoelectric Voltages and Zero Offsets

Voltages develop in electrical conductors with temperature gradients when no current is allowed to flow (thermal EMF's). Thermoelectric voltages appear when dissimilar metals are joined and joints are held at different temperatures. Typical thermoelectric voltages in cryogenic measurement systems are on the order of microvolts.

A zero offset is the signal value measured with no input to the measuring instrument. The zero offset can drift with time or temperature and is usually included in the instrument specifications.

Thermoelectric voltages and zero offsets can be eliminated from voltage measurements on ohmic resistors by reversal of the excitation current and use of the formula:

$$V = (V_+ - V_-)/2 \quad (7)$$

where V_+ and V_- are the voltages with respectively positive and negative excitation currents. Alternating current (ac) excitation can also be used with ohmic sensors to eliminate zero offsets.

Measurements made in rapid succession might not allow time for current switching and the required settling times. The error can be reduced by measuring the offset before and after a series of rapid measurements and subtracting the offset voltage from the measured voltages. The sum of the thermoelectric voltages and zero offset can be calculated as

$$V_o = (V_+ + V_-)/2 \quad (8)$$

Note that the resolution of V_o is practically limited by the resolution of the measurement system. The value of V_o can be expected to vary little in a static system, but may change during a thermal transient under study. The value of V_o should be rechecked as often as is practical.

The offset voltage V_o is best measured by reversing the current through a resistor. Measurement of V_o with zero excitation current is also possible, but large resistances can produce excessive time constants for discharge of any capacitances in the circuit, requiring long waiting times before V_o can be measured accurately.

Measurements on diodes do not allow current reversal. The value of V_o can be estimated by shorting the leads at the diode and measuring the offset voltage with zero excitation current at operating temperature.

Ground Loops and Electromagnetic Noise

Improper grounding of instruments or grounding at multiple points can allow current flows which result in small voltage offsets. One common problem is the grounding of cable shields at both ends. The current flow through ground loops is not necessarily constant, resulting in a fluctuating error voltage.

Electromagnetic pickup is a source of additional noise. Alternating current noise is a serious problem in sensors with nonlinear current-voltage characteristics (6). Measurement of the ac noise across the terminals of the reading instrument can give a quick indication of the magnitude of this noise source (thermal noise will be included in this measurement). Books on grounding and shielding can help to identify and eliminate both ground loops and electromagnetic noise (7,8).

Self Heating

Heat dissipated within a temperature sensor causes its temperature to rise, resulting in an error relative to the sensor's surroundings. Self heating errors might not affect relative temperature measurements. Attempting to correct for self heating errors by calculation or extrapolation is not considered good practice. An estimate of the self heating error should be included in the total error calculation instead. An easy way to check for self heating is to increase the power dissipation and check for an indicated temperature rise. Unfortunately, this procedure will not work with diodes. An indication of the self heating error can be made by reading the diode temperature in both a liquid bath and in a vacuum at the same temperature, as measured by a second thermometer not dissipating enough power to self heat significantly.

Calibration Uncertainty

Commercially calibrated temperature sensors should have calibrations traceable to international standards. Calibration uncertainties for sensors calibrated by Lake Shore are provided later in this paper. The calibration uncertainty of the temperature sensor must be included in accuracy calculations.

Interpolation Errors

Once a calibration has been performed, an interpolation function is required for temperatures which lie between calibration points. The interpolation method must be chosen with care since some fitting functions can be much worse than others. Common interpolation methods include linear interpolation, cubic splines and Chebychev polynomials. Formulas based on the physics of the sensor material may give the best fits when few fit parameters are used.

Use of an interpolation function adds to the temperature measurement uncertainty. The additional uncertainty due to an interpolation function can be gauged by the ability of the interpolation function to reproduce the calibration point temperatures from the calibration point resistances. Lake Shore calibration reports include the mean and largest deviations. Fitting with Chebychev polynomials is standard practice. Each calibration can be broken up into several ranges to decrease the fitting errors. Typical errors introduced by the interpolation function are on the order of one-tenth the calibration uncertainty.

CALIBRATION SYSTEM EXAMPLE

The example to be discussed in detail is the cryogenic temperature calibration facility operated by Lake Shore. This facility is designed to calibrate a variety of resistance and diode temperature sensors over the temperature range of 1.2 to 330 K.

Physical Construction

Calibrations are performed by mounting sensors on a probe to be inserted in a liquid helium cryostat (see Figure 3). The sensors are mounted in a gold-plated OFHC copper calibration block which provides an isothermal environment. Special adapters and a variety of calibration blocks allow calibration of sensors with varying shapes and sizes. The electrical leads from the sensors are soldered to contacts thermally anchored to a second gold-plated OFHC copper block directly above the calibration block. The thermal anchoring block is attached to a flange, on top of which is a liquid helium subpot. Surrounding the thermal anchoring and calibration blocks is an isothermal OFHC copper shield. The shield has a resistance wire heater wound around the outside with several layers of super-insulation overwrap to reduce thermal radiation to or from the vacuum can. Thermoelectric voltages are minimized by using continuous wire from the thermal anchoring block to the low thermal EMF connectors at the top of the probe which is at room temperature.

Operation

During cooldown, a small amount of helium gas is introduced into the vacuum chamber to act as a transfer medium. The cryostat is then filled with liquid helium and the calibration

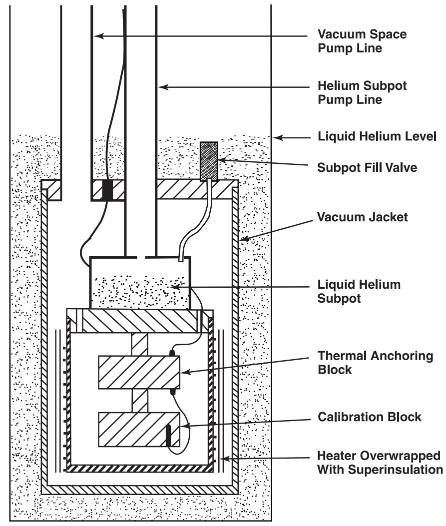

Figure 3. Calibration cryostat schematic

block and surrounding chamber cool to a nominal temperature of 4.2 K. The transfer gas is then pumped out. To obtain temperatures below 4.2 K, the subpot is filled with liquid helium and vacuum pumped. As the vapor pressure of the helium liquid in the subpot decreases, the temperature decreases. The pumping is controlled by a high resolution pumping valve. The subpot bath temperature is not actively controlled. Depending on the pumping speed and base pressure, temperatures as low as 1.05 K can be reached. To obtain temperatures above 4.2 K, the subpot is pumped dry and the heater is energized by the temperature controller. A diode monitors the nominal temperature of the isothermal shield and calibration block and the temperature is read by the temperature controller. The heater is used to bring the temperature to a point just below the desired temperature. The heater power is then reduced so that the temperature is increasing on the order of a millikelvin per minute. Data are taken when the drift rate is sufficiently small (typically about 10 minutes).

Electronic Equipment

The electronic equipment used in this facility consists of a HP3456A voltmeter, a Keithley model 224 variable current source, five Lake Shore model 8085 scanners, a Lake Shore DRC-82C temperature controller, five Guildline 9330 standard resistors (10 Ω, 100 Ω, 1 kΩ, 10 kΩ and 100 kΩ values), a 1000 Ω germanium standard thermometer and a 100 Ω platinum standard thermometer. Other electronic equipment such as the computer used for system control has no effect on the accuracy of the system. A block diagram of the equipment connection scheme is shown in Figure 4. Data acquisition is computer controlled. Two scanners are used to switch between each of twenty unknown sensors, one scanner is used to place one of the standard resistors into the circuit, one scanner chooses between the germanium and platinum standard, and the last scanner chooses whether the voltmeter measures the voltage drop across the unknown sensor or the standard resistor.

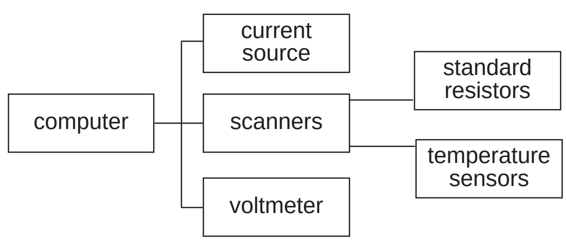

Figure 4. Thermometer calibration facility instrumentation block diagram

Resistance Measurements

The resistance of a sensor is measured by comparison with a standard resistor. Long term stability of resistor standards tends to be somewhat better than the long term stability of current sources, so overall accuracy is improved over methods relying on a calibrated current source.

The normal operating procedure is to place a resistor standard in series with the sensor whose resistance is to be measured. A voltmeter reading is taken with current in both the forward and reverse directions across the sensor. Voltmeter readings are then taken with current in both the forward and reverse directions across the standard resistor. The resistance of the sensor can be calculated using the relation

$$R_{sensor} = \frac{(V_+ - V_-)_{sensor}}{(V_+ - V_-)_{standard}} \times R_{standard} \qquad (9)$$

where V_+ and V_- are the voltages measured with current in the forward and reverse directions respectively. Measuring and averaging voltage for current in both forward and reverse directions serves two purposes: errors due to thermoelectric voltages are eliminated and voltmeter offsets are canceled out. In this situation, the voltmeter transfer specification, rather than the absolute measurement specification, applies. The gain in accuracy is about a factor of ten over using the voltmeter as an absolute measurement device.

Diode Measurements

Diode measurements are no more difficult to perform but typically less accurate. The reduced accuracy is a consequence of the nonlinear current-voltage characteristic of diodes. The voltage across the diode can be measured only in the forward direction, so the voltmeter must now make an absolute measurement. Without current reversal, thermoelectric voltages and voltmeter offsets may be present and these directly affect the achievable accuracy. The long-term accuracy and stability of the current source is also a factor. Fortunately, the small dynamic resistance reduces the error due to small current errors by a factor of 100 to 1000 (6).

Calibration

Calibration is accomplished by comparison calibration against standard thermometers. Two standard thermometers are used: a germanium resistance thermometer for the 1 to 28 K range and a platinum resistance thermometer for the 28 to 330 K range. A standard sensor reading is taken before and after every unknown sensor reading. The initial and final readings are averaged to compensate for temperature drifts between the time the standard and unknown are read.

Total System Accuracy Calculation

The attainable accuracy for a temperature measurement system depends on a number of variables. Lake Shore bases its calibrations on a calibrated voltmeter and calibrated working resistance standards to transfer a temperature scale from working temperature standards to unknown resistance temperature sensors. Calculating the total system accuracy requires information such as absolute and transfer specifications for equipment being used and a derating schedule for the calibration of the equipment. Some of this information is normally supplied with the equipment, but other parts are not. The manufacturer is the best source for this information. Keep in mind, however, that the degradation of the equipment is directly dependent upon its use and treatment.

Our voltmeters are calibrated every six months to ensure they meet their transfer specifications. Primary standard resistors are calibrated once per year. The working resistance standards are calibrated every six months against the primary standard resistors.The following table lists typical uncertainties for the 10 Ω, 100 Ω, 1000 Ω and 10 kΩ working standard

Nominal Value of Working Standard Resistor R (Ω)	1 Year Base Uncertainty of Primary Standard Resistor A	Voltmeter Transfer Accuracy B	Error From Possible Room Temperature Fluctuations C	Total Uncertainty for Working Standard Resistor MP	WC
		-	-		
100	15	6	5	17	26
1000	15	6	5	17	26
10000	20	6	5	22	31

Table I. Uncertainty estimates for calibrations of working standard resistors. Errors and uncertainties are expressed in parts per million (±ppm). Typical values are calculated by quadrature (MP) and worst case (WC) values by direct summation.

T (K)	CGR-1-1000 (mK)	GR-200A-1000 (mK)	PT-103 (mK)
1.5	1	1	-
4.2	1	1	-
10.	3	2	-
20.	10	6	14
30.	19	10	5
50.	41	20	3
100.	110	76	6
300.	425	-	16

Table II. Temperature measurement uncertainties in millikelvin for carbon glass (CGR), germanium (GR) and platinum (PT) sensors.

T [*K*]	<10	20	30	50	100	300
ε_T [mK]:	5	10	15	15	15	20

Table III. Uncertainties in realizing the ITS-90 temperature scale at the Lake Shore calibration facility.

	CGR-1-1000 [mK]		GR-200A-1000 [mK]		PT-103 [mK]	
T (K)	MP	WC	MP	WC	MP	WC
1.5	4	5	4	5	-	-
4.2	4	5	4	5	-	-
10.	4	5	4	5	-	-
20.	10	20	8	15	15	25
30.	20	35	12	25	10	20
50.	30	55	20	35	10	20
100.	65	125	45	90	10	20
300.	250	450	-	-	20	35

Table IV. Total temperature measurement uncertainties relative to ITS-90 in millikelvin for carbon glass (CGR), germanium (GR) and platinum (PT) sensors.

resistors. Uncertainties arise from shifts in the primary standard resistances, limitations of the voltmeter as a transfer device, and dependence of the voltmeter and standard resistors on variations in room temperature.

The total errors from the standard resistors due to calibration shifts and operating temperature variations are listed in Table I in terms of parts per million (ppm). The uncertainty estimates can be converted into equivalent temperature uncertainties given the temperature and specific sensitivity of the sensor measured using Equation 2.

Using the voltmeter as a transfer standard gives an improved accuracy over using it to make absolute measurements. The transfer accuracy of the voltmeter is roughly ±10 counts which translates to about ±1 µV on an absolute scale in the millivolt range. Signals for carbon glass and germanium sensors are kept between 1 and 3 mV so this is equivalent to a relative accuracy, ε_{rel}, of about 0.05%. Platinum sensors are read at a power somewhat less than 10 µW and produce voltage signals ranging from 3.5 mV at 30 K (1 mA current) to 27.5 mV at 300 K (0.25 mA). The voltmeter relative accuracy for 100 Ω platinum sensors ranges from 0.03% at 30 K to about 0.0036% at 300 K. Higher accuracy at higher temperatures is also observed in rhodium-iron sensors. Equivalent temperature uncertainties are given in Table II for a typical carbon glass resistor (model CGR-1-1000), germanium resistor (model GR-200A-1000) and a platinum resistor (model PT-103). The uncertainties due to calibration transfer of the resistance standards and that of the voltmeter transfer accuracies have been added together in this table.

Another important source of error comes from the error limits assigned to the secondary temperature standards calibrated by national standards laboratories. Based on estimates given in NBS Monograph 126 concerning the accuracy of the fixed points maintained at NIST (National Institute for Standards and Technology, formerly NBS) and the variations observed in platinum thermometers, an uncertainty estimate of ±5 mK can be made. Added to this uncertainty is the measurement uncertainty from Table II. Germanium standards (1000 Ω) are used below 28 K and platinum standards (100 Ω) are used above 30 K. The measurement uncertainty added to the calibration uncertainty of the secondary temperature standards gives the overall uncertainty in realizing the ITS-90 temperature scale. The uncertainty of Lake Shore calibrations relative to ITS-90 is given in Table III at several temperatures. The temperature resolution of the Lake Shore Calibration Facility is generally a factor of 10 or more better than our accuracy specification.

The total error of a given calibration is the combination of the first three tables. The total error is given in Table IV for the same representative temperature sensors included in Table II. The total uncertainty is expressed as millikelvin deviation from ITS-90. Two columns are given for each sensor. The "MP" column is the estimated most probable error of a given calibration computed using summation by quadrature. The "WC" column is the unlikely worst case error computed by direct summation of all error sources.

CONCLUSION

The accuracies stated apply only to the sensors as calibrated. An end user must be careful to distinguish between the desired measurement accuracy and the calibration accuracy of the sensor alone. Errors introduced by the user's measurement system, rough handling and inadequate thermal contact will add to the calibration uncertainty.

An estimate of the accuracy of a temperature sensor can be made by combining the errors due to calibration, interpolation and the measurement system. Errors can be added in quadrature to give the most probable error, or can be summed directly to give worst case error.

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